Exercise 44

Use logarithmic differentiation to find the derivative of the function.

$$y = x^{\cos x}$$

Solution

Take the natural logarithm of both sides and use the properties of logarithms to simplify the right side.

$$\begin{aligned}
\ln y &= \ln x^{\cos x} \\
&= \cos x \ln x
\end{aligned}$$

Differentiate both sides with respect to x.

$$\begin{aligned}
\frac{d}{dx}(\ln y) &= \frac{d}{dx}(\cos x \ln x) \\
\frac{1}{y} \cdot \frac{d}{dx}(y) &= \left[\frac{d}{dx}(\cos x)\right] \ln x + \cos x \left[\frac{d}{dx}(\ln x)\right] \\
\frac{1}{y} \cdot \frac{dy}{dx} &= (-\sin x) \ln x + \cos x \left(\frac{1}{x}\right) \\
\frac{1}{y}\frac{dy}{dx} &= -\sin x \ln x + \frac{\cos x}{x} \\
\frac{dy}{dx} &= y\left(\frac{\cos x - x \sin x \ln x}{x}\right) \\
&= x^{\cos x}\left(\frac{\cos x - x \sin x \ln x}{x}\right) \\
&= \frac{\cos x - x \sin x \ln x}{x^{1-\cos x}}
\end{aligned}$$